6.3-6.4: Decision Algorithms, Combinations and Permutations

Question 1. At a restaurant you can choose among 8 chicken dishes, 10 beef dishes, 4 seafood dishes and 12 vegetarian dishes. How many total options does this give you?

Addition Principle. When choosing among r disjoint alternatives, suppose that

$$\begin{aligned} &\text{alternative 1 has } n_1 \text{ possible outcomes} \\ &\text{alternative 2 has } n_2 \text{ possible outcomes} \\ &\qquad\qquad\qquad\qquad\qquad\qquad \cdots \\ &\text{alternative } r \text{ has } n_r \text{ possible outcomes} \end{aligned}$$

with no two of these outcomes the same. Then the total number of possible outcomes is $n_1 + n_2 + \cdots + n_r$.

Question 2. Now suppose you are at a restaurant and you can choose among 5 appetizers, 34 main dishes and 10 desserts. How many total meal options (including one appetizer, one main dish and one dessert) can you choose from?

Multiplication Principle. When making a sequence of choices with r steps, suppose that

$$\begin{aligned} &\text{step 1 has } n_1 \text{ possible outcomes} \\ &\text{step 2 has } n_2 \text{ possible outcomes} \\ &\qquad\qquad\qquad\qquad\qquad\qquad \cdots \\ &\text{step } r \text{ has } n_r \text{ possible outcomes} \end{aligned}$$

and that each sequence of choices results in a distinct outcome. Then the total number of possible outcomes is $n_1 \cdot n_2 \cdot \cdots \cdot n_r$.

Decision Algorithms.

Example 1. At an ice cream parlor you can choose between 15 flavors of ice cream and 5 flavors of frozen yogurt. In addition, you can choose to add one of ten toppings to your ice cream or yogurt and you can choose among 3 different sizes of cones for your ice cream and 2 different sizes of cups for your yogurt. How many different deserts can you choose from?

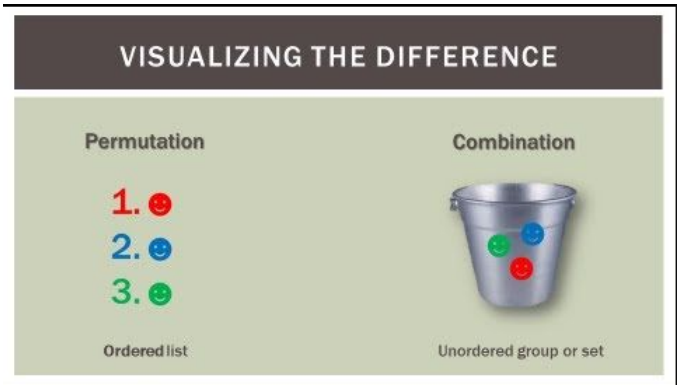
Example 2. You are playing Scrabble and losing. In your hand you have the letters k, e, r and e to work with. You wish to play all four letters to get yourself back into a manageable position. You can think of any four-letter words with this combination of letters, so you decide to list all possibilities. How large is your list?

Question 3. Your nasty English teacher (nasty compared to your awesome math teacher) is making five students give a speech in class. None of the five wish to go first so the teacher will have to choose the order they give their speeches. How many possible orderings of the five students can the teacher choose?

Definition 1. We call an ordered list of items a permutation of those items.

Question 4. There are one hundred people running in a race. However, the podium can only hold three people (first, second and third). How many different ways can the podium be ordered?

Permutations and Combinations. Definition 2. A permutation of n items taken r at a time is an *ordered* list of r items chosen from n. A combination of n items taken r at a time is an *unordered* set of r items chosen from n.



Question 5. Go back to question 4. There are still one hundred people in the race and still only three can stand on the podium. However, this time you are not worried about the order of the people on the podium, you are only worried about the collection of people on the podium. How many different combinations of people can there be?

Example 3. Calculate $C(11, 3)$ and $C(11, 8)$. Do you notice anything about these two numbers? Can you give a reason why what you noticed must be true?

Example 4. A bag contains three red marbles, three purple marbles, three green marbles and two yellow marbles (all of which are distinguishable from one another.)

(a) How many sets of four marbles are possible?

(b) How many sets of four are there such that each one is a different color?

(c) How many sets of four are there in which at least two are red?

(d) How many sets of four are there in which none are red, but at least one is green?